Scaling in urban systems: The CO_2 emissions case

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GSS-Green Growth

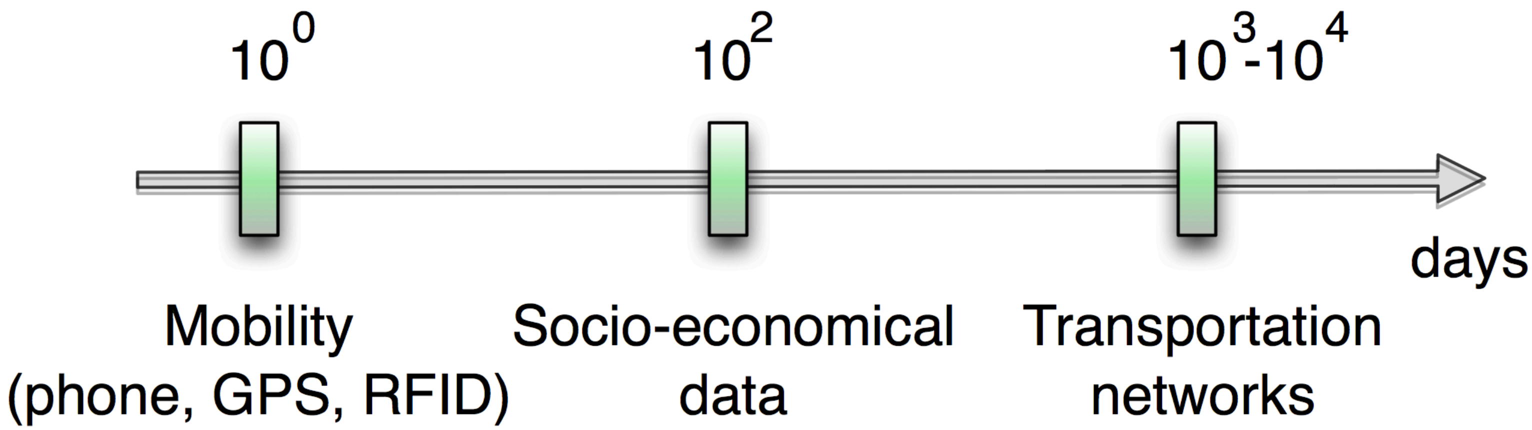
Long term goal: 'quantitative urbanism', or a new 'Science of Cities'

- Fundamental aspects
  - Understanding quantitatively the main mechanisms of urbanization
  - Modeling the evolution of urban systems

- Practical aspects
  - Urban planning
  - Hints for smarter cities
  - Green growth

Towards a (new) science of cities

- What changed ? Always more data about cities !
- Different scales, different phenomena



1. Spatial structure of the city (polycenters)

2. Scaling (with population)

3. Evolution of networks (roads and transportation)

Data and urban science

- Data availability
  - Large volumes
  - On many aspects: socio-economics, crime, health, …
  - For many, different cities in the world

- Data is good
  - Stylized facts
  - Test models and theories

- An important case: scaling of aggregated quantities

$$Y \sim P^{\beta}$$

Pumain, 2004

Scaling

$$Y \sim P^{\beta}$$

Table 1. Scaling exponents for urban indicators vs. city size

Y	β	95% CI	Adj-R^2	Observations	Country–year
New patents	1.27	[1.25,1.29]	0.72	331	U.S. 2001
Inventors	1.25	[1.22,1.27]	0.76	331	U.S. 2001
Private R&D employment	1.34	[1.29,1.39]	0.92	266	U.S. 2002
"Supercreative" employment	1.15	[1.11,1.18]	0.89	287	U.S. 2003
R&D establishments	1.19	[1.14,1.22]	0.77	287	U.S. 1997
R&D employment	1.26	[1.18,1.43]	0.93	295	China 2002
Total wages	1.12	[1.09,1.13]	0.96	361	U.S. 2002
Total bank deposits	1.08	[1.03,1.11]	0.91	267	U.S. 1996
GDP	1.15	[1.06,1.23]	0.96	295	China 2002
GDP	1.26	[1.09,1.46]	0.64	196	EU 1999–2003
GDP	1.13	[1.03,1.23]	0.94	37	Germany 2003
Total electrical consumption	1.07	[1.03,1.11]	0.88	392	Germany 2002
New AIDS cases	1.23	[1.18,1.29]	0.76	93	U.S. 2002–2003
Serious crimes	1.16	[1.11, 1.18]	0.89	287	U.S. 2003
Total housing	1.00	[0.99,1.01]	0.99	316	U.S. 1990
Total employment	1.01	[0.99,1.02]	0.98	331	U.S. 2001
Household electrical consumption	1.00	[0.94,1.06]	0.88	377	Germany 2002
Household electrical consumption	1.05	[0.89,1.22]	0.91	295	China 2002
Household water consumption	1.01	[0.89,1.11]	0.96	295	China 2002
Gasoline stations	0.77	[0.74,0.81]	0.93	318	U.S. 2001
Gasoline sales	0.79	[0.73,0.80]	0.94	318	U.S. 2001
Length of electrical cables	0.87	[0.82,0.92]	0.75	380	Germany 2002
Road surface	0.83	[0.74,0.92]	0.87	29	Germany 2002

Bettencourt et al, PNAS 2007

Scaling of CO2 (transport related) emissions

- Two sources of errors in

$$Q_{CO_2} = F(P)$$

in estimating Q_{CO2} and in estimating P

- For transport related emissions, Q is proportional to the time spent - not distance ! (Glaeaser & Kahn, 2010)

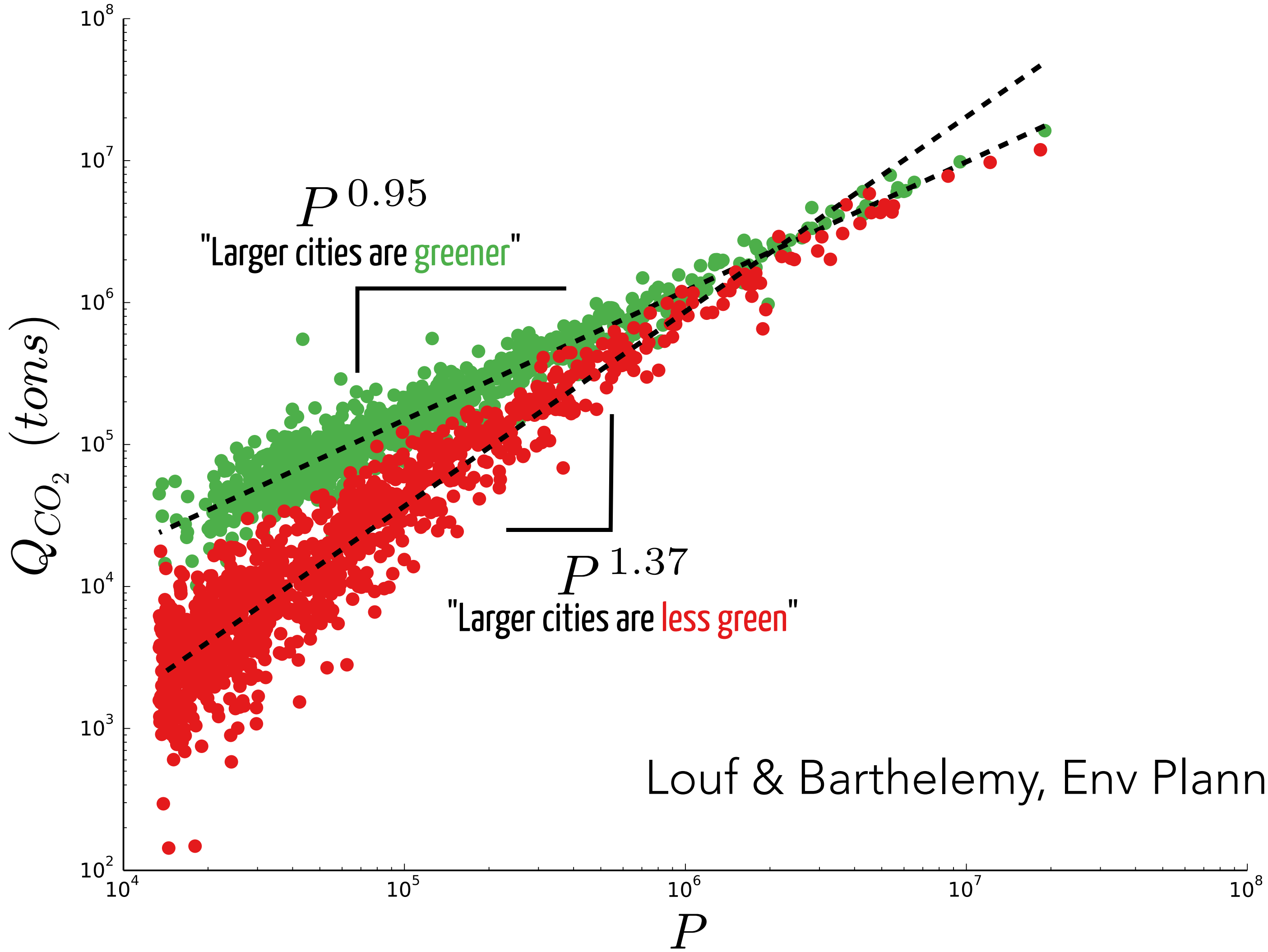
- Estimating P is tricky (Arcaute et al, 2013)

Louf & Barthelemy, Env Plan B, in press

Scaling of CO2 emissions: summary

Paper	City definition	CO2 estimate	Result
Glaeser, Kahn 2008	MSA/IPUMS metro definition	Miles driven (NHTS)	Sublinear $\beta < 1$
Fragkias et al 2013	MSA	Vulcan Project 1999-2008 10km x 10km	Sublinear $\beta = 0.93$
Ribsky et al 2014	MSA	Various sources GHG emissions	Sublinear (USA) β versus GDP
Oliveira et al 2014	CCA MSA	Vulcan Project	$\beta = 1.38$ $\beta = 0.92$

Data is good but….



Louf & Barthelemy, Env Plann B, in press

- Scaling of CO2 transport-related emission
- Same dataset but at different aggregation levels !

Data is not enough !

- Data provides some scaling and can confirm theoretical ideas and…

- Models are needed in order to progress !

- We need 'physical' models
  - Keeping economical ingredients
  - Minimal number of parameters
  - Out-of-equilibrium models

- Testing predictions against data

- We illustrate this on the example of mobility and CO_2 emissions

Mobility is the key

- We need to model how individuals move from home to work
  - Connected to the spatial structure of the city
  - Once known allows to compute all mobility/transport related quantities

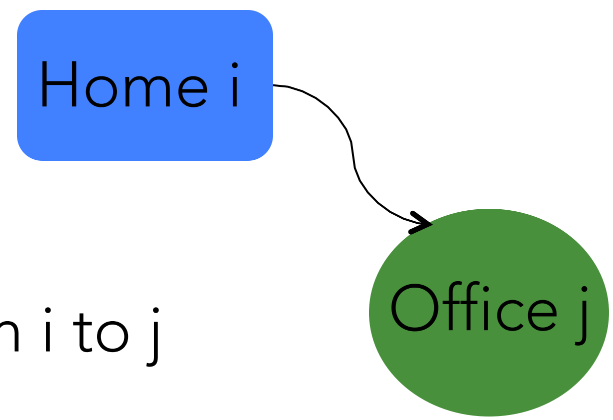
- Problem largely studied in geography, and in urban economics: Fujita-Ogawa model (1982)

Spatial economics: Fujita-Ogawa (1982)

- A model for the spatial structure of cities: an agent will choose to live in *i* and work in *j* such that

$$Z_0(i,j) = W(j) - C_R(i) - C_T(i,j)$$

is maximum



- W(j) is the wage at j
- C_R(i) is the rent at i
- C_T(i,j) is the transportation cost from i to j [proportional to d(i,j)]

- … and a similar equation for companies (profit)
- Optimization for all individuals and all companies

Spatial economics: Fujita-Ogawa (1982)

- There are many problems with this model:

  - Not dynamical: optimization. We want an out-of-equilibrium model

  - No congestion (!) We want to include congestion (for car traffic)

  - No empirical test. Extract testable predictions

    (see the book: Spatial Economics, by Fujita, Krugman, Venables)

'Dynamical random Fujita-Ogawa': Results

Quantity	Theoretical dependence on P ($\delta = \alpha/\alpha + 1$)	Predicted value	Measured value
A/ℓ^2	$\left(\frac{P}{c}\right)^{2\delta}$	$2\delta = 0.78 \pm 0.20$	0.853 ± 0.011 ($r^2 = 0.93$) [USA]
L_N/ℓ	$\sqrt{P}\ \left(\frac{P}{c}\right)^{\delta}$	$\frac{1}{2} + \delta = 0.89 \pm 0.10$	0.765 ± 0.033 ($r^2 = 0.92$) [USA]
$\delta\tau/\tau$	$P\ \left(\frac{P}{c}\right)^{\delta}$	$1 + \delta = 1.39 \pm 0.10$	1.270 ± 0.067 ($r^2 = 0.97$) [USA]
$Q_{gas,CO_2}/\ell$	$P\left(\frac{P}{c}\right)^{\delta}$	$1 + \delta = 1.39 \pm 0.10$	1.262 ± 0.089 ($r^2 = 0.94$) [USA] 1.212 ± 0.098 ($r^2 = 0.83$) [OECD]

- Polycentric structure
- Qualitative agreement ! (super- versus sublinear)
- The CO2 emissions scale superlinearly !
- For large P: Congestion becomes too large

=> large cities based on congestion-sensitive modes (cars, ..) are not economically sustainable !

Louf, MB, PRL 2013, Sci Rep 2014

Discussion (1): data

- Empirical scalings rarely constitute a proof by themselves (noise, arbitrariness in the fit,…), but constitute a support for models !

- From the same dataset, contradictory results !
  - Need for harmonized databases (definition of the city!)
  - Available to all scientists

Discussion (2): models

- We need to construct solid scientific foundations for models
  - Hierarchy of mechanisms
  - Keeping economical ingredients
  - Minimal number of parameters
  - Out-of-equilibrium models
- Testing predictions against data
- Modeling the city ? Or some aspects of ?

Discussion (3): interdisciplinarity

- Interdisciplinary effort !
- Communication difficulties between different communities working on these problems
- Discussing and revisiting urban economics…

Thank you for your attention.

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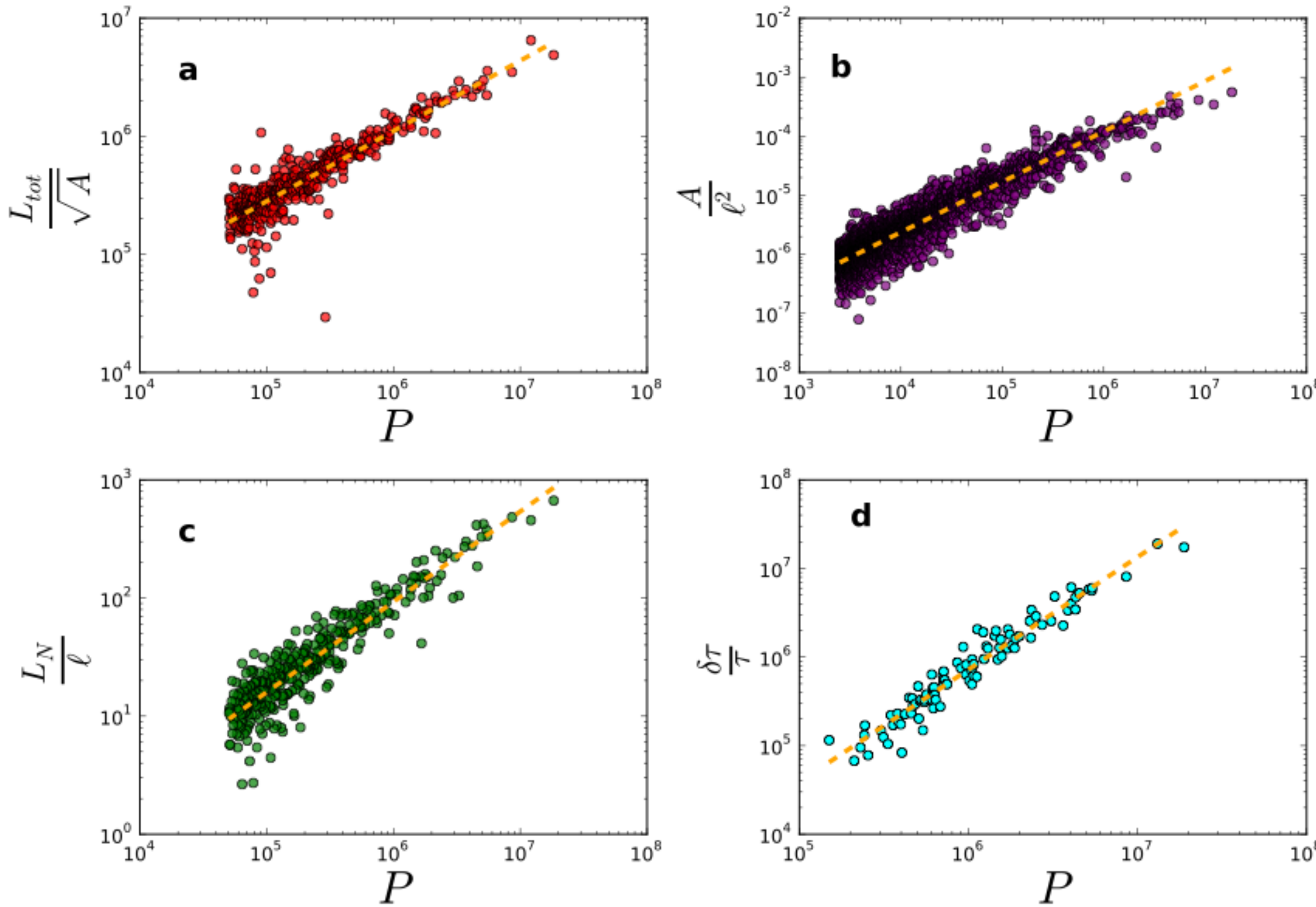
Additional slides

Mobility related measures



441 US cities+ some OECD data (Louf & MB, PRL 2013; Sci Rep 2014)

Fujita-Ogawa revisited

- Every time step, add a new individual at a random i

- The individual will choose to work in *j* (among N_c possible centers) such that

$$Z(i,j) = \eta_j - \frac{d_{ij}}{\ell}\left[1 + \left(\frac{T(j)}{c}\right)^{\mu}\right]$$

is maximum

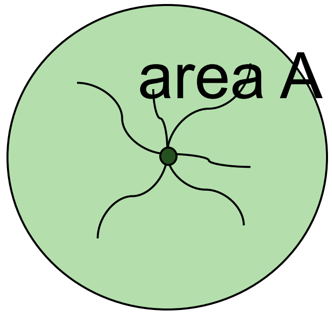
- Not a global optimization

- W(j) is the wage at j --> random

- C_T(i,j) is the transportation cost from i to j: depends on the traffic from i to j --> congestion effects

Louf & MB, PRL 2013

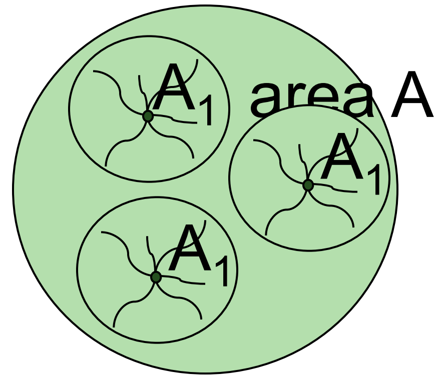
Naive scaling: Total commuting distance



Monocentric

$$\ell_1 \sim \sqrt{A}$$

$$L_{tot}/\sqrt{A} \sim P$$



Nearest neighbor

$$\ell_1 \sim 1/\sqrt{\rho} \sim \frac{\sqrt{A}}{\sqrt{P}}$$

$$L_{tot}/\sqrt{A} \sim P^{1/2}$$

- In general, we can then expect

$$\frac{L_{tot}}{\sqrt{A}} \sim P^{\beta} \quad \beta \in [0.5, 1]$$

Naive scaling: Total commuting distance

- Another argument:

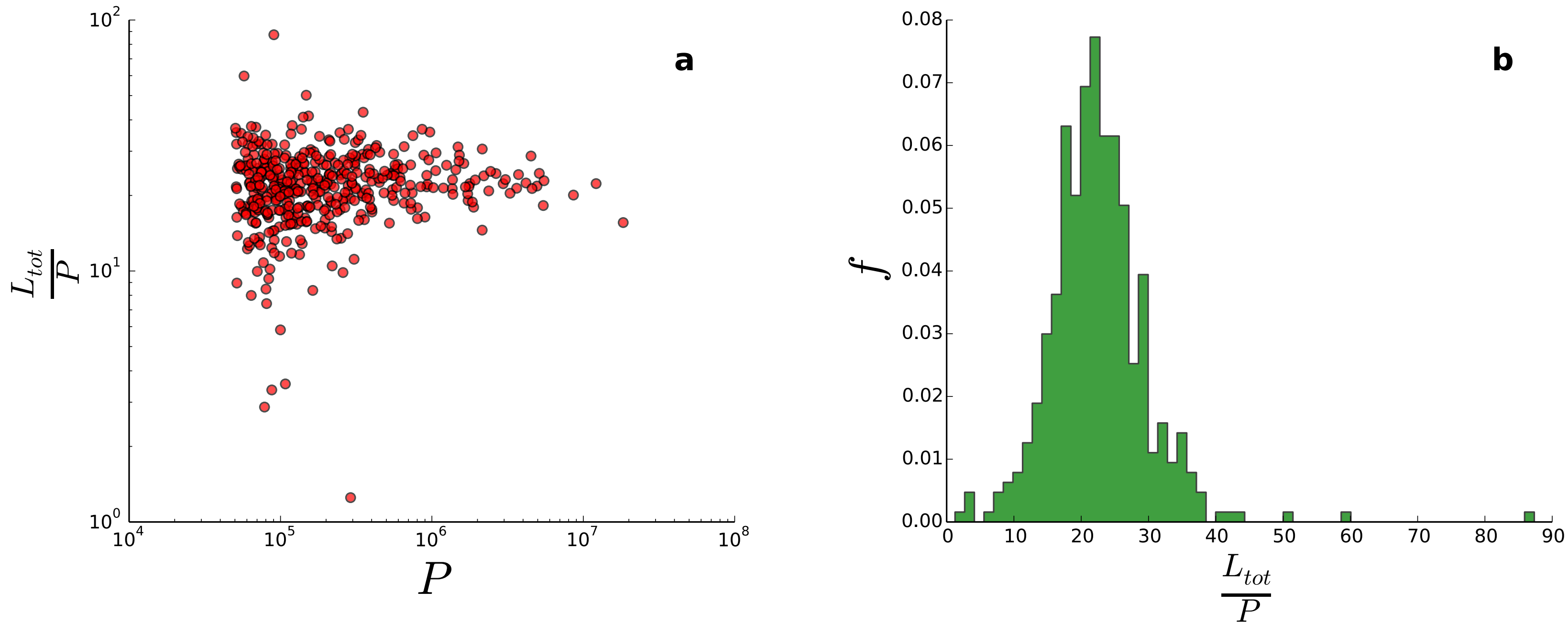
$$\frac{L_{tot}}{P} \sim \text{const.}$$

- Simple consistency relation

$$\begin{cases} A \sim P^{\alpha} \\ L_{tot}/\sqrt{A} \sim P^{\beta} \end{cases}$$

$$\Rightarrow 1 - \frac{\alpha}{2} = \beta$$

Scaling in cities: measures



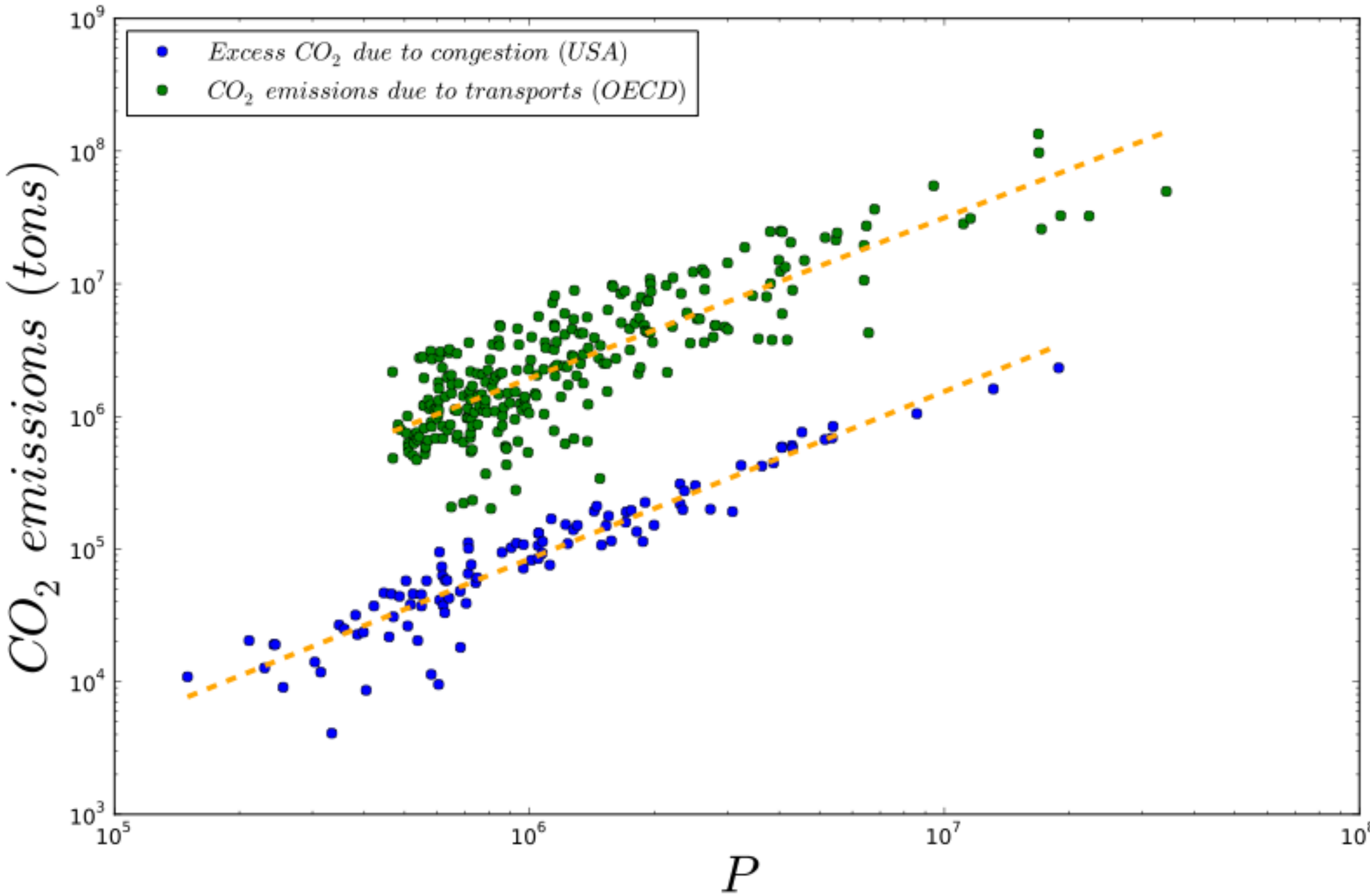
$$\frac{L_{tot}}{P} \sim const.$$

US cities+ some OECD data (Louf, MB, 2013)

Scaling in cities: measures

Rybski et al (2013)
Makse et al (2014)
Louf & MB (2014)

- CO_2 emissions (transport related)



- Superlinear !

Scaling in cities: measures

Quantity	"Naive" scaling	Measured value for the exponent of P
$L_{tot}/\sqrt{A}$	$1/2, 1$	0.595 ± 0.026 ($r^2 = 0.91$) [USA]
L_{tot}/P	0	0.03 ± 0.02 ($r^2 = 0.1$) [USA]
$L_N/\sqrt{A}$	$1/2$	0.42 ± 0.03($r^2 = 0.83$) [USA]
A/ℓ^2	1	0.853 ± 0.011 ($r^2 = 0.93$) [USA]
$\delta\tau/\tau$	?	1.270 ± 0.067 ($r^2 = 0.97$) [USA]

- We have consistency: $1 - \frac{0.853}{2} = 0.574 \simeq 0.595$

- L_{tot} seems to scale as P

- Area A? Monocentric picture seems wrong

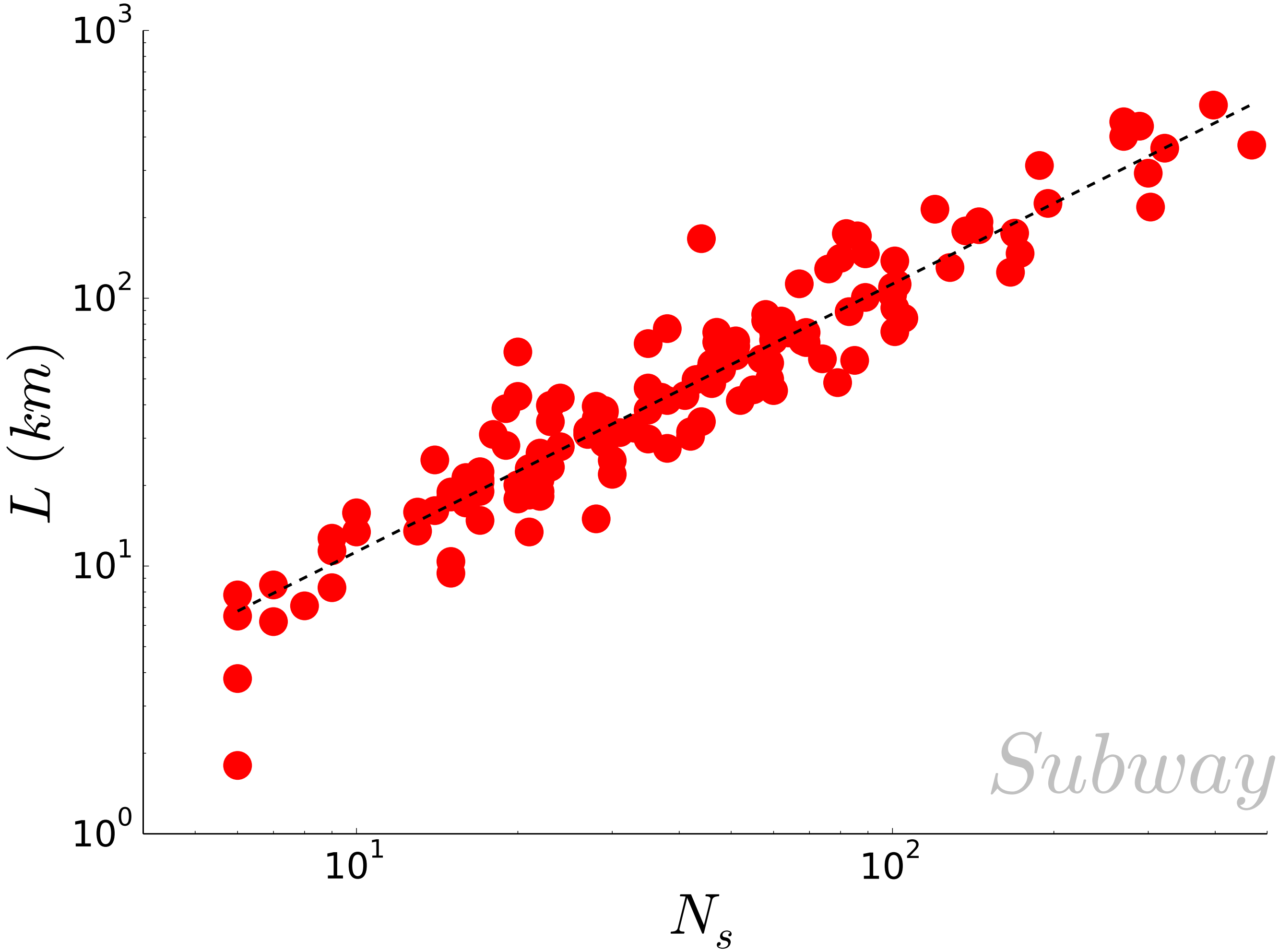
Example 2. Scaling in transportation networks

How scale the number of stations, the total length, the ridership ?

Louf, MB PLoS One (2014)

Empirical analysis

- In general, scalings are no proof, but can support theoretical ideas and predictions



Empirical analysis

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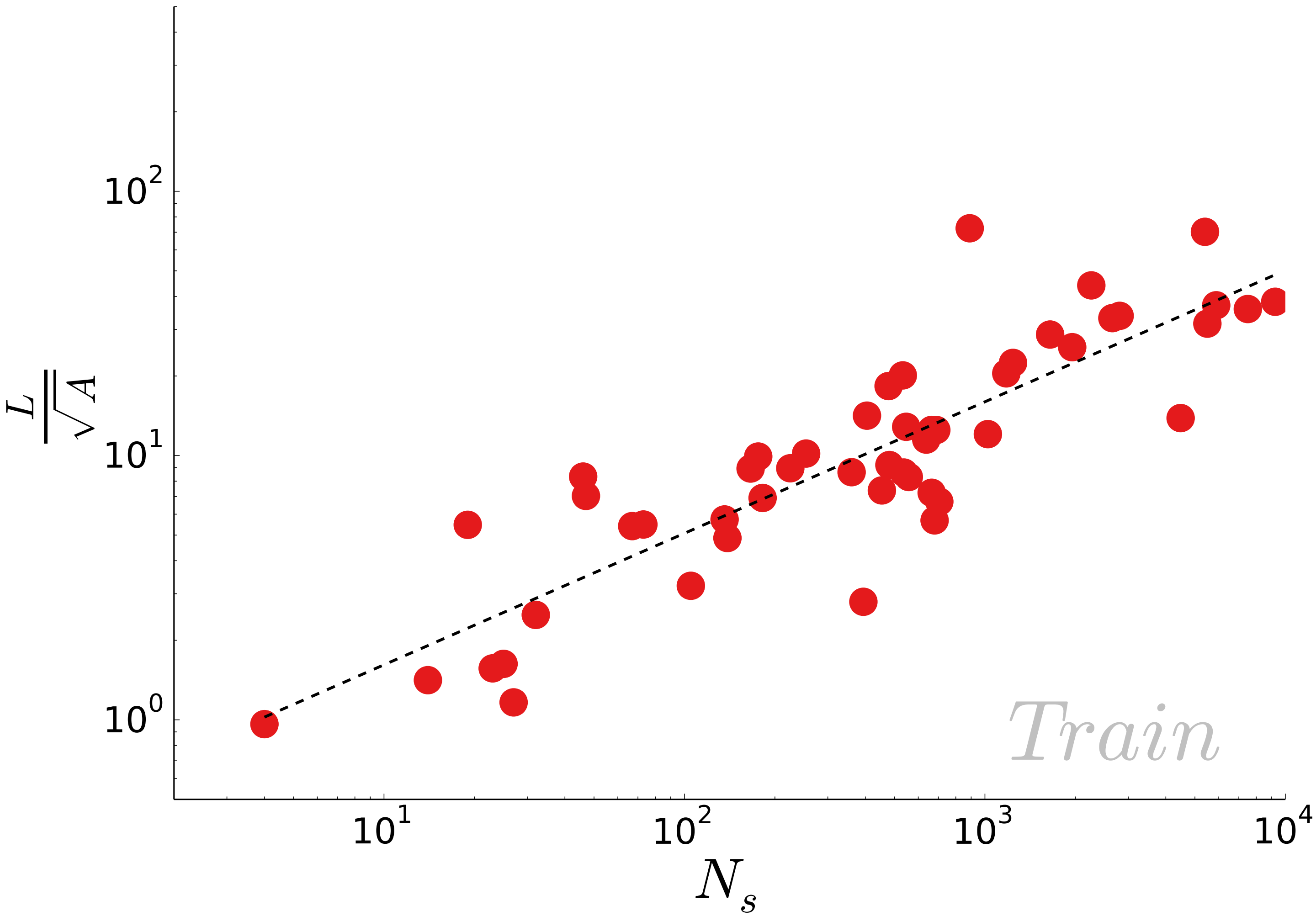

General framework

- Iterative growth model such that

$$Z_e = B_e - C_e$$

is maximum (B_e is the benefit of constructing edge e and C_e its cost)

- Total budget $Z = \sum_e B_e - C_e = B - C$

B: total expected benefits
C: total cost (mainly due to maintenance)

- Stationary state assumption for mature networks

$$Z \approx 0$$

Subways and railways

$$Z_{\text{subway}} \simeq Rf - \epsilon_L L - \epsilon_S N_S$$

- R: total ridership (per year); f: ticket price
- ϵ_L: maintenance cost per unit length (per year)
- ϵ_S: maintenance cost per station (per year)

$$R_i \simeq \xi_i \frac{N_S}{A} \pi d_0^2$$

$$d_0 \simeq 500m \quad \ell_1 = 2d_0 \simeq 1km$$

Subways and railways

$$Z_{\text{train}} \simeq T f_L - \epsilon_L L$$

- T: total traveled distance (per year)
- f_L: ticket price (per unit length)
- ϵ_L: maintenance cost per unit length (per year) $\gg \epsilon_S N_S$
- $$L \simeq N_S \sqrt{\frac{A}{N_S}} \simeq \sqrt{A N_S}$$
- $$\begin{aligned} Z_{train} &\approx 0 \\ T &\simeq R\ell_1 \\ L &= N_S \ell_1 \end{aligned} \quad \Rightarrow R \simeq \frac{\epsilon_L N_s}{f_L}$$

Summary of results

- We obtain results in agreement with empirical scalings

	Subway	**Train**
L/N_s	cste.	$\sqrt{\frac{A}{N_s}}$
R	$\frac{P}{A} N_s$	N_s
G	N_s	L

- Fundamental difference between subways and railways: -
  - subways: interstation distance seems to be constant and determined by the typical walking distance
  - for railways: the interstation distance scales with the number of stations.